Description Logics - Basic Notions

Camilo Thorne

Room 00.012
Institut für Maschinelle Sprachverarbeitung
Universität Stuttgart
+49 (0) 711 685-81369
camilo.thorne@ims.uni-stuttgart.de

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Syntax ($\mathcal{ALCHQI}$)

In the $\mathcal{ALCHQI}$ description logic, concepts C and roles R are defined according to the following syntax:

$$\begin{array}{rcl} C & ::= & A \mid \exists_{\leq k} R : C \mid \neg C \mid C \sqcap C \\ R & ::= & r \mid r^{-} \end{array}$$

where A stands for an atomic concept or concept name (a unary predicate), r for a role name (a binary predicate) and r^{-} for its inverse.

We can enrich the set of $\mathcal{ALCHQI}$ concepts via

$$\begin{array}{rclcrcl} \bot & = & \neg A \sqcap \neg A & \quad & C \sqcup C' & = & \neg(\neg C \sqcap \neg C') \\ \exists_{\geq k} R : C & = & \neg(\exists_{\leq k+1} R : \neg C) & & \exists R & = & \exists R : \top \\ \exists R : C & = & \exists_{\geq 1} R : C & & \forall R : C & = & \neg(\exists R : \neg C) \\ \exists_{= k} & = & \exists_{\leq k} R : C \sqcap \exists_{\geq k} R : C & & \top & = & \neg \bot \end{array}$$

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Knowledge Bases ($\mathcal{ALCHQI}$)

A TBox $\mathcal{T}$, is a set of assertions of the form

- concept inclusions of the form $C \sqsubseteq C'$, stating IS-A (set inclusion) between concepts C and C'
- role inclusions of the form $R \sqsubseteq R'$, stating IS-A (set inclusion) among roles

An ABox, expressing extensional knowledge, is a set $\mathcal{A}$ of membership assertions of the form $C(c)$, $R(c, c')$, where c, c' are individual constants

A knowledge base is a tuple $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ an ABox

Interpretations ($\mathcal{ALCHQI}$)

The semantics of concepts, assertions, ontologies and knowledge bases is specified by considering first-order interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where the interpretation function $\cdot^{\mathcal{I}}$ maps

1. concept names A and roles r into, resp., subsets $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ of the domain and $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ of its cross product, and
2. object names c to elements $c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ of the domain

It can be extended to complex concepts C and roles R by structural recursion

Models, Entailment ($\mathcal{ALCHQI}$)

An interpretation $\mathcal{I}$ is said to be a model of a concept inclusion $C \sqsubseteq C'$ (resp., role inclusion $R \sqsubseteq R'$ or membership assertions $C(c)$, $R(c, c')$), in symbols $\boxed{\mathcal{I} \models C \sqsubseteq C'}$, when

$$C \sqsubseteq C' \quad \text{(resp.,} \quad R^{\mathcal{I}} \sqsubseteq R'^{\mathcal{I}}, \quad c^{\mathcal{I}} \in C^{\mathcal{I}}, \quad (c^{\mathcal{I}}, c'^{\mathcal{I}}) \in R^{\mathcal{I}})$$

It is said to be a model of TBox $\mathcal{T}$ (resp. ABox $\mathcal{A}$), in symbols $\boxed{\mathcal{I} \models \mathcal{T}}$, when it is the model of its assertions

It is said to be a model of a knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, in symbols $\boxed{\mathcal{I} \models \mathcal{K}}$, when it is a model of $\mathcal{T}$ and $\mathcal{A}$

Finally, an (ABox or TBox) assertion α is entailed by knowledge base $\mathcal{K}$, in symbols $\boxed{\mathcal{K} \models \alpha}$, iff every interpretation $\mathcal{I}$ that is a model of $\mathcal{K}$ is also a model of α

Summary ($\mathcal{ALCHQI}$)

Syntax	Semantics
c	$c^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
$\exists_{\geq k} R : C$	$(\exists_{\geq k} R : C)^{\mathcal{I}} := \{c \mid \lvert\{c' \text{ s.t. } (c, c') \in R^{\mathcal{I}} \text{ and } c' \in C^{\mathcal{I}}\}\rvert \geq k\}$
$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$C \sqcap C'$	$(C \sqcap C')^{\mathcal{I}} = C^{\mathcal{I}} \cap C'^{\mathcal{I}}$
r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
r^{-}	$(r^{-})^{\mathcal{I}} = \{(d, d') \mid (d', d) \in r^{\mathcal{I}}\}$
$C \sqsubseteq C'$	$\mathcal{I} \models C \sqsubseteq C'$ iff $C^{\mathcal{I}} \subseteq C'^{\mathcal{I}}$
$R \sqsubseteq R'$	$\mathcal{I} \models R \sqsubseteq R'$ iff $R^{\mathcal{I}} \subseteq R'^{\mathcal{I}}$
$C(c)$	$\mathcal{I} \models C(c)$ iff $c^{\mathcal{I}} \in C^{\mathcal{I}}$
$R(c, c')$	$\mathcal{I} \models R(c, c')$ iff $(c^{\mathcal{I}}, c'^{\mathcal{I}}) \in R^{\mathcal{I}}$
$\mathcal{T}$	$\mathcal{I} \models \mathcal{T}$ iff for all $\alpha \in \mathcal{T}, \mathcal{I} \models \alpha$
$\mathcal{A}$	$\mathcal{I} \models \mathcal{A}$ iff for all $\alpha \in \mathcal{A}, \mathcal{I} \models \alpha$
$(\mathcal{T}, \mathcal{A})$	$\mathcal{I} \models (\mathcal{T}, \mathcal{A})$ iff $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$

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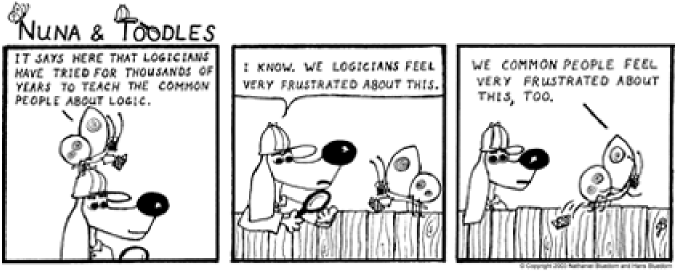

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