Studying the Distribution of Fragments of English Using Deep Semantic Annotation

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Outline

Motivation

- Suppose we came across the following argument in some text

  Every Italian loves pasta and football
  Silvio is Italian

  that entails

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- Common-sense reasoning

- How difficult can it be to reason over such arguments?

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- How often constructs with high semantic complexity occur in corpora?

⇒ Can we leverage on state-of-the-art deep semantic annotation?

QUE: Is semantic complexity inversely proportional to frequency?

The Fragments of English [PHT06]

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$\Rightarrow$ Efficient translation, but how costly is logical reasoning?

The Fragments of English [PHT06, Tho10]

Fragment	Coverage	Fo Operators
COP($\neg$)	Copula ("is a"), nouns ("man"), intransitive verbs ("runs"), "every", "some" names ("Joe"), adjectives ("thin") (+"not")	$\{\forall, \exists, (\neg)\}$
COP($\neg$)+TV	COP($\neg$) +transitive verbs ("loves")	$\{\forall, \exists, (\neg)\}\cup$
COP($\neg$)+DTV	COP($\neg$) +ditransitive verbs ("gives")	$\{\forall, \exists, (\neg)\}\cup$
COP($\neg$)+TV +DTV	COP($\neg$)+TV + ditransitive verbs	$\{\forall, \exists, (\neg)\}\cup$
COP($\neg$)+Rel	COP($\neg$)+ ("who", "that", "which") "and", intersective adjectives (+"or")	$\{\forall, \exists, \wedge, (\neg, \vee)\}$
COP($\neg$)+Rel +TV	COP($\neg$)+Rel +transitive verbs	$\{\forall, \exists, \wedge, (\neg, \vee)\}$
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The Fragments of English (Examples)

Fragment	**Example**	**Fo**
COP	Every politician cheats	$\forall x(\mathit{Politician}(x) \rightarrow \mathit{Cheat}(x))$
COP$^{\neg}$	Some philosopher is not trustworthy	$\exists x(\mathit{Philosopher}(x) \wedge \neg \mathit{Trusted}(x))$
COP$^{\neg}$+TV	John does not love Luke	$\neg \mathit{Loves}(\text{John}, \text{Luke})$
COP+TV +DTV	John gives a book to Jane Some man likes every candy	$\exists x \mathit{Book}(x) \wedge$ $\mathit{Gives}(\text{John}, x, \text{Jane})$ $\exists x(\mathit{Man}(x) \wedge$ $\forall y \mathit{Candy}(x) \rightarrow \mathit{Likes}(x, y))$
COP +Rel	Every idiot who is a philosopher cheats	$\forall x(\mathit{Idiot}(x) \wedge \mathit{Philosopher}(x)$ $\rightarrow \mathit{Cheat}(x))$
COP$^{\neg}$ +Rel	Some man who does not cheat is trustworthy	$\forall x(\mathit{Man}(x) \wedge \neg \mathit{Cheat}(x)$ $\rightarrow \mathit{Trusted}(x))$
$\vdots$	$\vdots$	$\vdots$

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$\Rightarrow$ People have a hard time processing intractable English constructs [Szy09]

$\Rightarrow$ What about corpora?

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     What is one common element of major religions?

     Boxer outputs

$$\exists y \exists z \exists e \exists u(\texttt{card}(y,u) \wedge \texttt{c1num}(u) \wedge \texttt{nnumeral1}(u) \wedge \texttt{acommon1}(y) \wedge \texttt{nelement1}(y) \wedge \texttt{amajor1}(z) \wedge \texttt{nreligions1}(z) \wedge \texttt{nevent1}(e) \wedge \texttt{rof1}(y,z))$$

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   - $\wedge$ and $\exists$ co-occur, but not $\vee$, $\neg$, or $\forall$

2. Does Boxer beat patterns?
   - for $\neg$: "not", "no"
   - for $\exists$: "some", "a"
   - for $\forall$: "all", "every, "each"
   - for $\wedge$: "who", "what", "which", "and"
   - for $\vee$: "or"

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(b) 4 "Boolean-closed" classes: $\{\exists, \wedge, \neg\}$, $\{\exists, \wedge, \neg, \forall\}$, $\{\exists, \wedge, \neg, \forall, \vee\}$ and $\{\neg, \forall\}$

$\Rightarrow$ Each class "approximates" a fragment of English

$\Rightarrow$ Study relationships between class frequency $fr(c)$ and class rank or expressivity $rk(c)$

Corpora

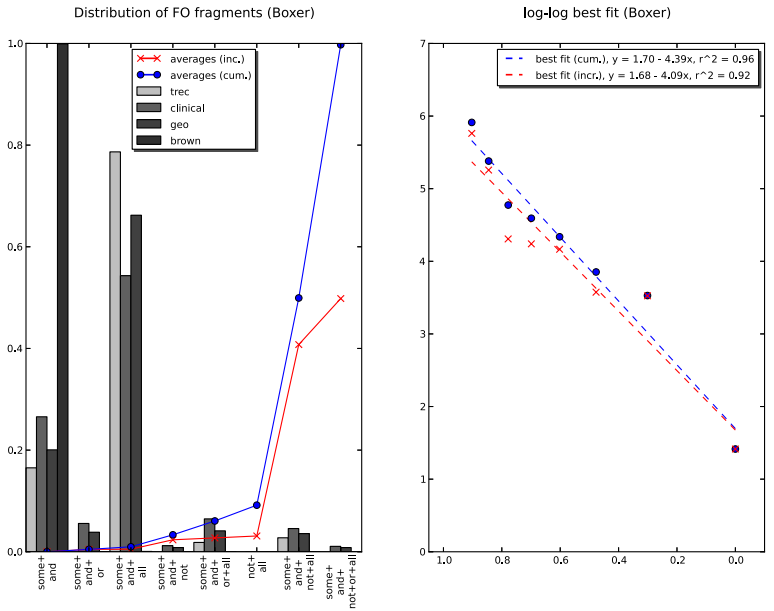
- We considered
  1. a subset (A: press articles) of the Brown corpus (http://nltk.googlecode.com/svn/trunk/nltk_data/index.xml)
  2. a subset (Geoquery880) of the Geoquery corpus (http://www.cs.utexas.edu/users/ml/nldata/geoquery.html)
  3. a corpus of clinical questions (http://clinques.nlm.nih.gov)
  4. a sample from the TREC 2008 corpus (http://trec.nist.gov)

- Of features

Corpus	Size	Domain	Type
Brown	19,741 sent.	Open (news)	Declarative
Geoquery	364 ques.	Geographical	Interrogative
Clinical ques.	12,189 ques.	Clinical	Interrogative
TREC 2008	436 ques.	Open	Interrogative

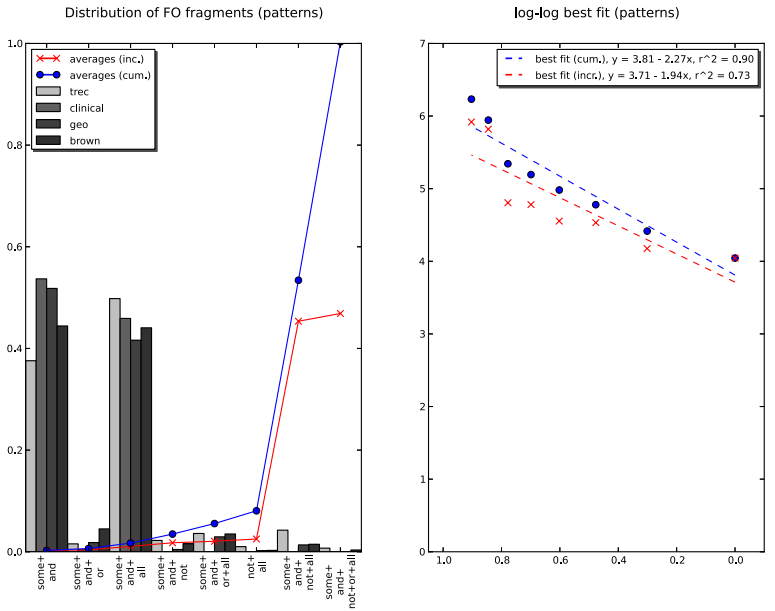
Power Law Fitting [Bar09]

Boxer



	(power law)	(R^2)
cum:	$fr(c) = \frac{1.7}{rk(c)^{4.39}}$	0.96
means:	$fr(c) = \frac{1.68}{rk(c)^{4.09}}$	0.92

Patterns



	(power law)	(R^2)
cum:	$fr(c) = \frac{3.81}{rk(c)^{2.27}}$	0.90
means:	$fr(c) = \frac{3.71}{rk(c)^{1.94}}$	0.72

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	Ent.	Rel. Ent.	Skewness	χ^2	p-value	df.
Boxer	1.53	0.51	1.93	293731473.0	0.0	7
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$\Rightarrow$ Data skewed towards simple classes

$\Rightarrow$ But... power law model inconclusive

$\Rightarrow$ We don't know Boxer's accuracy either

Conclusions and Further Work

1. We have experimented with a methodology based on the Boxer semantic parser
2. The distribution obtained may seem to indicate that "non-Boolean-closed" (tractable) fragments occur more often than "Boolean-closed" (intractable) fragments
3. The results obtained thus far remain inconclusive

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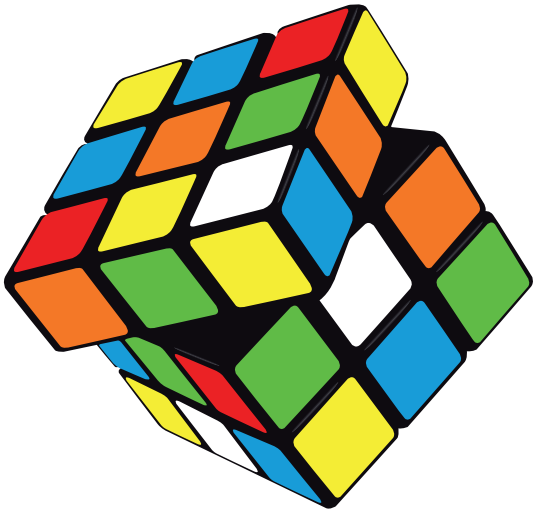
2. The distribution obtained may seem to indicate that "non-Boolean-closed" (tractable) fragments occur more often than "Boolean-closed" (intractable) fragments
3. The results obtained thus far remain inconclusive
4. To counter these shortcomings we would like in the future to
   - run the experiment with other deep semantic parsers (e.g., minimal recursion semantics [Cop07]),
   - consider larger corpora, in particular declarative corpora
   - consider more involved statistical tests



Thank you :-)

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Semantic Complexity [PHT06, Tho10]

	Combined	**Data**
COP	in LSPACE	in LSPACE
COP+TV	in PTIME	in LSPACE
COP+DTV	in PTIME	in LSPACE
COP+TV+DTV	in PTIME	in LSPACE
COP+Rel	PTIME-complete	in LSPACE
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COP+Rel+DTV+TV	PTIME-complete	in PTIME

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$\text{COP}^{\neg}$+Rel	NPTIME-complete	in LSPACE
$\text{COP}^{\neg}$+Rel+TV	EXP-complete	NPTIME-complete
$\text{COP}^{\neg}$+Rel+DTV	NEXP-complete	NPTIME-complete
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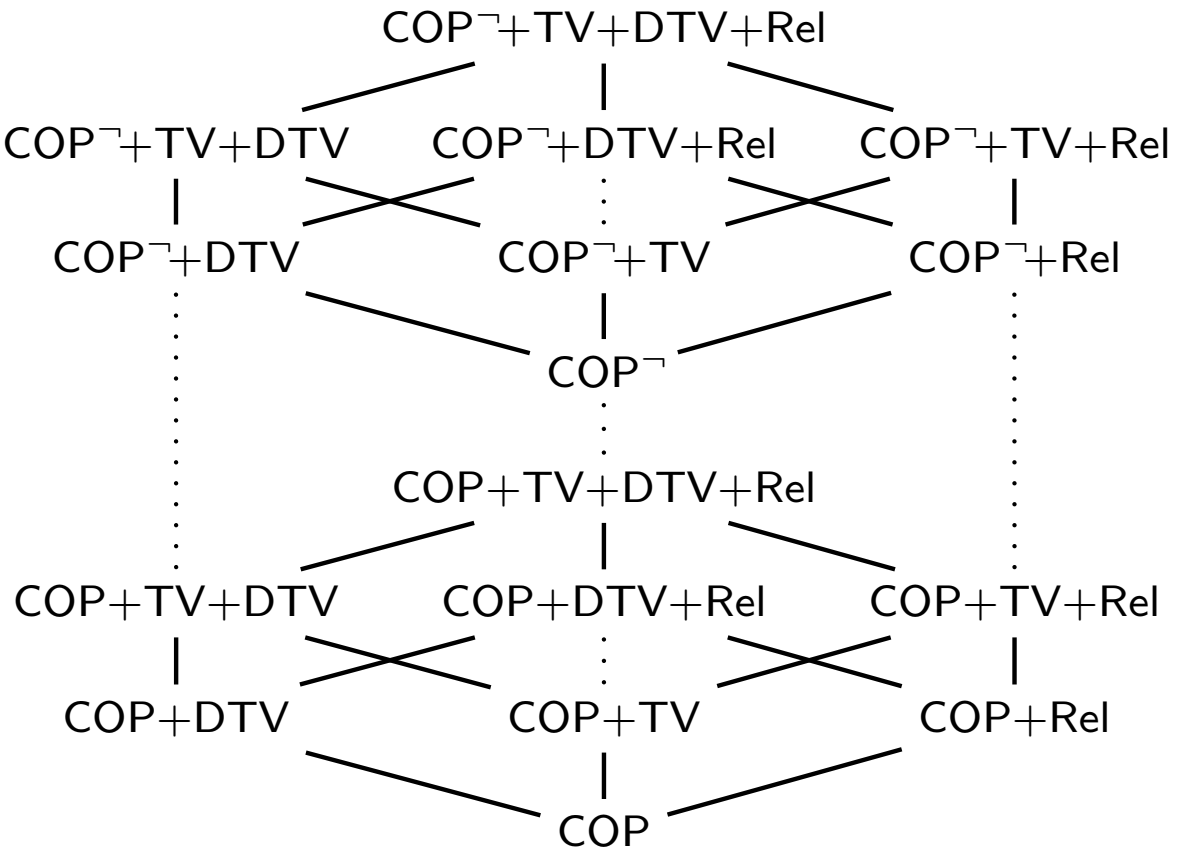
$\text{COP}^{\neg}$+Rel+DTV+TV	NEXP-complete	NPTIME-complete

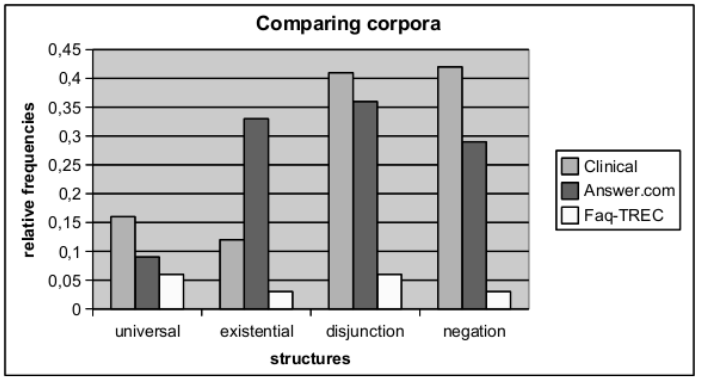
The Fragments of English (Expressive Power)



Linguistic Patters in Questions [BBC+07] (Related)

The analysis presented can be compared to the more linguistics-based methodology followed in [BBC+07]

The frequency distribution of classes of logical word patterns such as “all”, “both”, “each”, “every”, “everybody”, “everyone”, “any”, “none”, “nothing” was analyzed

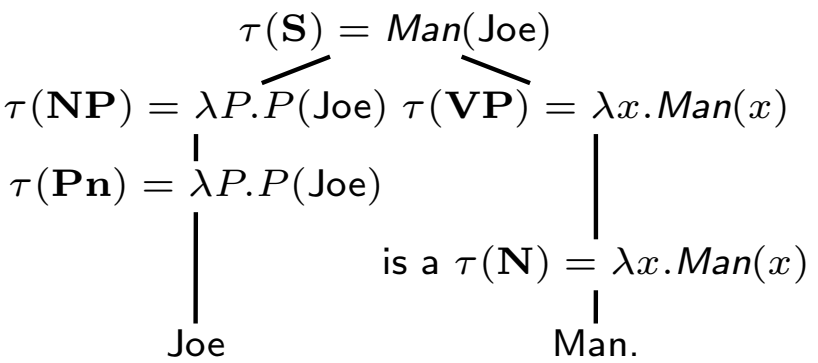


Only interrogative corpora were analyzed and co-occurence disregarded

The Fragment COP($\neg$) (Example)

Phrase Structure Rules	
$\mathbf{S} \rightarrow \mathbf{NP\ VP}$	$\tau(\mathbf{S}) = \tau(\mathbf{NP})(\tau(\mathbf{VP}))$
$\mathbf{VP} \rightarrow$ is a $\mathbf{N}$	$\tau(\mathbf{VP}) = \tau(\mathbf{N})$
$\mathbf{VP} \rightarrow$ is $\mathbf{Adj}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Adj})$
$\mathbf{VP} \rightarrow \mathbf{IV}$	$\tau(\mathbf{VP}) = \tau(\mathbf{IV})$
$\mathbf{NP} \rightarrow \mathbf{Pn}$	$\tau(\mathbf{NP}) = \tau(\mathbf{Pn})$
$\mathbf{NP} \rightarrow \mathbf{Det\ N}$	$\tau(\mathbf{NP}) = \tau(\mathbf{Det})(\tau(\mathbf{N}))$
$(\mathbf{VP} \rightarrow$ is $\mathbf{Ng}$ a $\mathbf{N}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Ng})(\tau(\mathbf{N})))$
$(\mathbf{VP} \rightarrow$ does $\mathbf{Ng\ IV}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Ng})(\tau(\mathbf{IV})))$

Function Lexicon	
$\mathbf{Det} \rightarrow$ every	$\tau(\mathbf{Det}) = \lambda P.Q.\forall x(P(x) \rightarrow Q(x))$
$\mathbf{Det} \rightarrow$ some	$\tau(\mathbf{Det}) = \lambda P.Q.\exists x(P(x) \wedge Q(x))$
$(\mathbf{Ng} \rightarrow$ not	$\tau(\mathbf{Ng}) = \lambda P.\lambda x.\neg P(x))$



Every man left $\rightsquigarrow$
$\forall x(\mathit{Man}(x) \rightarrow \mathit{Leave}(x))$

$\vdots$

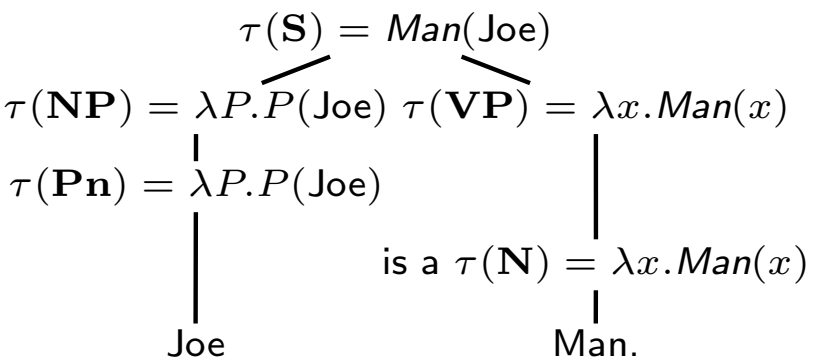
Some man is (not) a philosopher $\rightsquigarrow$
$\exists x(\mathit{Man}(x) \wedge (\neg)\mathit{Philosopher}(x))$

$\vdots$

The Fragment COP($\neg$) (Example)

Phrase Structure Rules	
$\mathbf{S} \rightarrow \mathbf{NP\ VP}$	$\tau(\mathbf{S}) = \tau(\mathbf{NP})(\tau(\mathbf{VP}))$
$\mathbf{VP} \rightarrow$ is a $\mathbf{N}$	$\tau(\mathbf{VP}) = \tau(\mathbf{N})$
$\mathbf{VP} \rightarrow$ is $\mathbf{Adj}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Adj})$
$\mathbf{VP} \rightarrow \mathbf{IV}$	$\tau(\mathbf{VP}) = \tau(\mathbf{IV})$
$\mathbf{NP} \rightarrow \mathbf{Pn}$	$\tau(\mathbf{NP}) = \tau(\mathbf{Pn})$
$\mathbf{NP} \rightarrow \mathbf{Det\ N}$	$\tau(\mathbf{NP}) = \tau(\mathbf{Det})(\tau(\mathbf{N}))$
$(\mathbf{VP} \rightarrow$ is $\mathbf{Ng}$ a $\mathbf{N}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Ng})(\tau(\mathbf{N}))$
$(\mathbf{VP} \rightarrow$ does $\mathbf{Ng\ IV}$	$\tau(\mathbf{VP}) = \tau(\mathbf{Ng})(\tau(\mathbf{IV}))$

Function Lexicon	
$\mathbf{Det} \rightarrow$ every	$\tau(\mathbf{Det}) = \lambda P.Q.\forall x(P(x) \rightarrow Q(x))$
$\mathbf{Det} \rightarrow$ some	$\tau(\mathbf{Det}) = \lambda P.Q.\exists x(P(x) \wedge Q(x))$
$(\mathbf{Ng} \rightarrow$ not	$\tau(\mathbf{Ng}) = \lambda P.\lambda x.\neg P(x))$



Every man left $\rightsquigarrow$
$\forall x(Man(x) \rightarrow Leave(x))$

$\vdots$

Some man is (not) a philosopher $\rightsquigarrow$
$\exists x(Man(x) \wedge (\neg) Philosopher(x))$

$\vdots$

Entropy and Relative Entropy (Reminder)

The entropy $H(X)$ of a (random) variable $X = x_1, \ldots, x_n$ is

$$H(X) = -\sum_i p(x_i) \cdot \log_2(p(x_i))$$

where $p(\cdot)$ is the p.m.f. of X; entropy measures how random the distribution of X is

The relative entropy $H_{rel}(X)$ of X, defined as

$$H_{rel}(X) = \frac{H(X)}{\log_2(n)}$$

and transforms $H(X)$ into a metric, since

$$\log_2(n) = \log_2(\max_i p(x_i))$$

Power Laws and Log-Log Regressions (Reminder)

We can transform power law models into linear models via logarithmic scaling

$$
\begin{aligned}
y &= \frac{b}{x^m} \\
&\Leftrightarrow \\
\log_{10}(y) &= \log_{10}(\tfrac{b}{x^m}) \\
&= \log_{10}(b) - \log_{10}(x^m) \\
&= \log_{10}(b) - m \cdot \log_{10}(x)
\end{aligned}
$$

If the R^2 coefficient is sufficiently high, we can say (with some caveats!!!) that the data (sample) is power law distributed

Linear Regression (Reminder)

A linear regression model has the form

$$Y = \theta X$$

with parameters $\theta = (m, b)^T$ (a gradient and an intercept)

It can be used to predict the value(s) of a dependent variable Y (e.g., frequency) vis-á-vis the value(s) of a predictor X (e.g., class complexity rank)

The least squares method computes the linear model whose parameters θ^* minimize square error

$$\theta^* = \arg\min_{\theta} \sum_i (y_i - \theta(x_i))^2$$

The R^2 coefficient provides a measure of confidence in the inferred model $Y = \theta^* X$ and is defined in terms of square error variance and dependent variable variance, i.e.,

$$R^2 = \frac{Var(\theta^* X)}{Var(Y)}$$