

# Aggregate Queries over Ontologies

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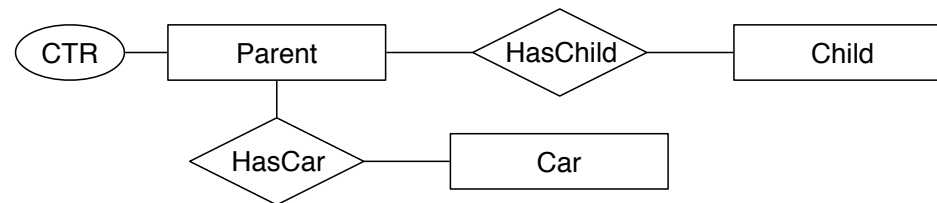
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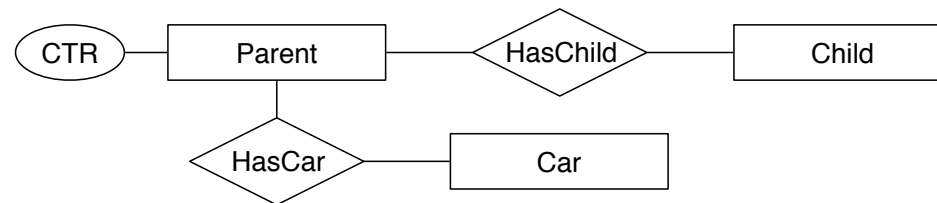
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Family models (partly) how families receive in Italy subsidies for their children, whose data can come from arbitrary sources

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Less is known, however, about DL or OWL ontologies

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5. Conclusions and further work

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DEF. By deleting aggregation functions and disregarding nestings we can transform an CAQ into its **core**

$$\tilde{q}(\vec{x}, \vec{y}) \leftarrow \phi, \psi.$$

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$$\begin{aligned} \text{Sat}_{\mathcal{D}}(\phi, \psi) &:= \{\gamma \mid \gamma \text{ satisfies } \phi \text{ and } \psi \text{ w.r.t. } \mathcal{D}\} \\ \text{Sat}_{\mathcal{D}}^{\vec{w}^\phi}(\phi, \psi) &:= \{\gamma \upharpoonright \vec{w}^\phi \mid \gamma \in \text{Sat}_{\mathcal{D}}(\phi, \psi)\} \end{aligned}$$

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DEF. The set of **answers** of  $q$  is defined as

$$q^{\mathcal{D}} := \{\langle \vec{d}, \alpha(G_{\vec{d}}) \rangle \mid \vec{d} = \gamma(\vec{x}) \text{ for some } \gamma \in \text{Sat}_{\mathcal{D}}^{\vec{w}^\phi}(\phi)\}$$

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Then: (i)  $G_{\bar{\epsilon}} = \{ \langle \text{Anna} \rangle \}$ , (ii)  $q_b^{\mathcal{D}_{\text{fam}}} = \{ \langle 1 \rangle \}$  and (iii)  $\tilde{q}_b^{\mathcal{D}_{\text{fam}}} = \{ \langle \text{Anna} \rangle \}$

## *DL-Lite* Ontologies

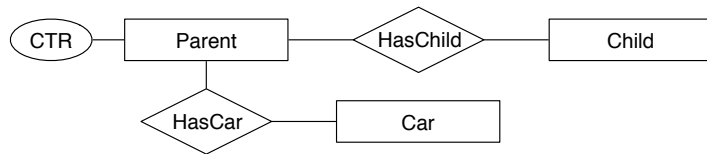
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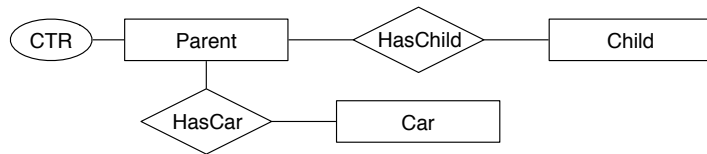
$Parent \sqsubseteq \exists CTR$	$\exists CTR \sqsubseteq Parent$
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$\exists hasCar^- \sqsubseteq Car$	$\exists hasChild^- \sqsubseteq Child$



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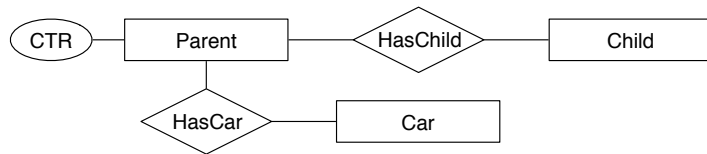
$$\begin{array}{ll}
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 \exists \text{hasCar}^- \sqsubseteq \text{Car} & \exists \text{hasChild}^- \sqsubseteq \text{Child} \\
 \exists \text{hasChild} \sqsubseteq \text{Parent} & \\
 \end{array}$$

Whereas  $\mathcal{D}_{\text{fam}}$  now stores data consistent with the  $\mathcal{T}_{\text{fam}}$  ontology:

Parent	CTR		hasCar	
PName	PName	Amount	PName	CType
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By combining them we get an OBDAS  $\langle \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}} \rangle$  over the family domain

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EXAMPLE. Consider as CQ (the core of  $q_b$ ) that asks for all parents that own cars:

$$\tilde{q}_b(x) \leftarrow \text{Parent}(x), \text{hasCar}(x, y).$$

Asking  $\tilde{q}_b$  to  $\langle \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}} \rangle$  gives as certain answers

$$\text{cert}(\tilde{q}_b, \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}}) = \{\langle \text{Anna} \rangle, \langle \text{Luisa} \rangle\}$$

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This does not carry on to AQs

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In different DBs, multiplicities can differ: consider the following two DBs  $\mathcal{D}'$  and  $\mathcal{D}''$  satisfying  $\langle \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}} \rangle$  where

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**THEO**. The certain answers of CAQs over OBDASs are, in general, empty

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$$q'_b(\mathbf{countd}(x)) \leftarrow \text{Parent}(x), \text{hasCar}(x, y)$$

In different DBs, multiplicities can differ: consider the following two DBs  $\mathcal{D}'$  and  $\mathcal{D}''$  satisfying  $\langle \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}} \rangle$  where

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Therefore:

$$\text{cert}(q_b, \mathcal{T}_{\text{fam}}, \mathcal{D}_{\text{fam}}) = \emptyset$$

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EAQs extend the syntax and semantics of AQs

## Epistemic Aggregate Queries (EAQs)

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Consider an ODBAS  $\mathcal{K} := \langle \mathcal{O}, \mathcal{D} \rangle$ ,  $\mathcal{D}'$  a DB s.t.  $\mathcal{D}' \models \mathcal{K}$  and an EAQ  $q(\vec{x}, \alpha(\vec{y})) \leftarrow \mathbf{K}\vec{x}, \vec{y}, \vec{z}. \phi, [\psi]$  with auxiliary query  $q_{aux}(\vec{x}, \vec{y}, \vec{z}) \leftarrow \phi, \psi$

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DEF. The **K satisfying assignments** and the **K restricted satisfying assignments** are defined as

$$\begin{aligned} \text{KSat}_{\mathcal{D}', \mathcal{K}}(\vec{w}; \phi, \psi) &:= \{\gamma \in \text{Sat}_{\mathcal{D}'}(\phi, \psi) \mid \gamma(\vec{w}) \in \text{cert}(q_{aux}, \mathcal{K})\} \\ \text{KSat}_{\mathcal{D}', \mathcal{K}}^{\vec{x} \cup \vec{y} \cup \vec{z}^\phi}(\vec{w}; \phi, \psi) &:= \{\gamma \upharpoonright \vec{x} \cup \vec{y} \cup \vec{z}^\phi \mid \gamma \in \text{KSat}_{\mathcal{D}', \mathcal{K}}(\vec{w}; \phi, \psi)\} \end{aligned}$$

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DEF. The **K answers** of  $q$  over  $\mathcal{D}'$  and the **epistemic certain answers** of  $q$  over  $\mathcal{K}$  are defined as

$$\begin{aligned} q(\mathcal{D}', \mathcal{K}) &:= \{\langle \vec{d}, \alpha(H_{\vec{d}}) \rangle \mid \gamma(\vec{x}) = \vec{d} \text{ for some } \gamma \in \text{KSat}_{\mathcal{D}', \mathcal{K}}^{\vec{x} \cup \vec{y} \cup \vec{z}^\phi}(\vec{w}; \phi, \psi)\} \\ \text{Ecert}(q, \mathcal{K}) &:= \bigcap \{q(\mathcal{D}', \mathcal{K}) \mid \mathcal{D}' \models \mathcal{K}\} \end{aligned}$$

# Epistemic Aggregate Queries (EAQs)

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EXAMPLE. Consider the EAQ  $q_b^k$  for **(b)**

$$q_b^k(\mathbf{countd}(x)) \leftarrow \mathbf{K}x, y. \textit{Parent}(x), \textit{hasCar}(x, y)$$

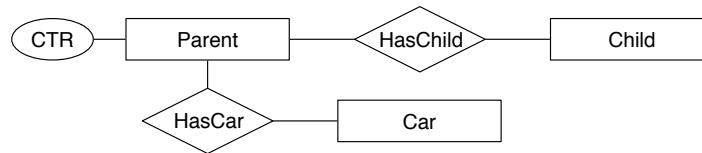
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That asks how many parents are known to own a car by the OBDAS  $\langle \mathcal{T}_{fam}, \mathcal{D}_{fam} \rangle$ :



Parent	CTR		hasCar	
PName	PName	Amount	PName	CType
Anna	Luisa	100	Anna	VwGolf
	Anna	150	Anna	FiatUno

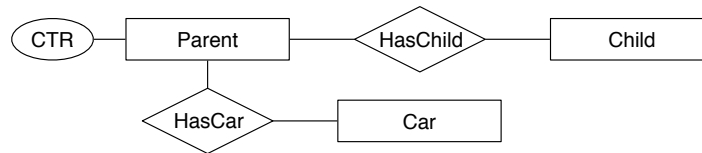
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Since we now restrict ourselves to information explicitly stated in the ODBAS, we get

$$H_{\bar{\epsilon}} = \{ \langle \text{Anna} \rangle, \langle \text{Anna} \rangle \}$$

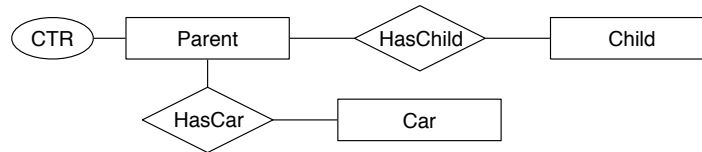
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Therefore,  $\mathbf{countd}(\{ \langle \text{Anna}, \text{Anna} \rangle \}) = 1$  and the epistemic certain answers are

$$ECert(q_b^k, \mathcal{T}_{fam}, \mathcal{D}_{fam}) = \{ \langle 1 \rangle \}$$

# The Generic Algorithm GA

---

**Input:** EAQ  $q(\vec{x}, \alpha(\vec{y})) \leftarrow \mathbf{K}\vec{x}, \vec{y}, \vec{z}. \phi, [\psi]$   
*DL-Lite* ODBAS  $\mathcal{K}$

**Output:** set of tuples  $O$



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**Do:** build  $q_{aux}$ :  $q_{aux}(\vec{x}, \vec{y}, \vec{z}) \leftarrow \phi, \psi$   
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build  $q_1$ :  $q_1(\vec{x}, \alpha(y)) \leftarrow v_0(\vec{x}, \vec{y}, \vec{z}^\phi)$   
compute certain answers:  $D_1 := cert(aux_q, \langle \mathcal{D}, \mathcal{T} \rangle)$   
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GA is sound and complete w.r.t.  $Ecertain(q, \mathcal{K})$  under certain conditions

# Properties

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Let  $\mathcal{K} := \langle \mathcal{T}, \mathcal{D} \rangle$  be an ODBA, let  $q(\vec{x}, \alpha(\vec{y})) \leftarrow \mathbf{K} \vec{x}, \vec{y}, \vec{z}$ .  $\phi, [\psi]$  be an EAQ and  $x, y \in \text{Var}(\phi, \psi)$

# Properties

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Restrictedness ensures non-triviality, soundness and completeness

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 $\text{Parent} \sqsubseteq \exists \text{hasCar}$          $\exists \text{hasCar} \sqsubseteq \text{Parent}$   
 $\exists \text{hasCar}^- \sqsubseteq \text{Car}$          $\exists \text{hasChild}^- \sqsubseteq \text{Child}$

Parent	CTR
PName	PName    Amount
Anna	Luisa    100
	Anna     150

hasCar
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Restrictedness guarantees non-emptiness with "intensional" aggregates

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THEO. (count)

THEO. (countd,min,max)

# Properties

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We would also like to study the problems of containment and equivalence of EAQs (i.e. optimization)