Computational Logic Lab IV

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DPLL. Given an set Γ of propositional unsatisfiable clauses, a *justification* of Γ w.r.t. a **SAT** solver $\mathrm{A}(\cdot)$ is the (finite) sequence (or history) $(r_1, \ldots, r_n)$ of rule applications r_i, $1 \leq i \leq n$, from which the empty clause $\perp$ was derived.

1. Modify an implementation of DPLL so that, when given an unsatisfiable set Γ of clauses, it provides a justification in terms of

    i. resolution applications, or
    ii. splitting applications.

2. If possible, extend the procedure so that it also returns the history of learning and/or backjumping applications.

BDDs. A set of propositional formulas Γ is said to be *minimal* if, whenever $\varphi \in \Gamma$, $\Gamma \setminus \{\varphi\} \not\models \varphi$.

1. Use BDDs to test for the minimality of a set Γ of propositional formulas.

2. Apply this algorithm to test for the minimality of

$$\Gamma \quad := \quad \{ \quad p \rightarrow (q \vee r), \quad p \vee (\neg q \wedge \neg r), \quad q \quad \}.$$

Stålmarck's Algorithm. Stålmarck's algorithm $\textsc{Stal}(\cdot)$ is another state-of-the art **SAT** solver that checks for the unsatisfiability of propositional formulas φ in negation normal form *nnf*(φ) by reasoning on their definitorial form *defi*(*nnf*(φ)).

Stålmarck's algorithm is known to outperform DPLL over Urquhart formulas: An *Urquhart formula* is a tautology of the form $\varphi_u := a_1 \Leftrightarrow a_2 \Leftrightarrow \ldots \Leftrightarrow a_n \Leftrightarrow a_1 \Leftrightarrow a_2 \Leftrightarrow \ldots \Leftrightarrow a_n$ for some $n \geq 1$. If φ_u has k biconditionals, then $\textsc{Stal}(\cdot)$ will stop and return **true** over $\neg\varphi_u$ in $\mathbf{O}(k^4)$ time[1].

1. Show that Urquhart formulas are 2-easy by Stålmarck's method.

2. Implement an OCaml function that generates an Urquhart formula from input parameter `n`, and compare the performance of the OCaml implementations of DPLL and Stålmarck's method on Urquhart formulas (possibly w.r.t. both time and space).

[1] Sample Urquhart formulas would be $p \Leftrightarrow q \Leftrightarrow p \Leftrightarrow q$ and $p \Leftrightarrow q \Leftrightarrow r \Leftrightarrow p \Leftrightarrow q \Leftrightarrow r$.