Computational Logic Lab III

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Propositional Logic - The Russian Spy Puzzle. Consider the following puzzle:

> Three diplomats, Steinberger, Blumenthal and Cohn, are attending a diplomatic meeting. We know that exactly one of them is Russian, and the other two German. We also know that every Russian is a spy. During the toast, Cohn says "you know, Steinberger, you are as German as I'm Russian". It is known that Cohn always speaks the truth when drinking. Is Blumenthal therefore a Russian spy?

We want to show, by means of **SAT** solving, that from this information it follows that Blumenthal *is not* a Russian spy.

SAT Solving. In general, *problems* **P** are sets of *instances* or *inputs* x satisfying a property/question. If $x \in \mathbf{P}$, we say that x is a *positive instance* of **P**, *negative* otherwise. Thereupon *problem solving* for a problem **P** consists in designing an algorithm $\textsc{Alg}(\cdot)$, called *solver*, s.t., for all instances x, $\textsc{Alg}(x) = \mathbf{true}$ iff $x \in \mathbf{P}$, i.e., that *solves* **P**.

Perhaps the most important class of problems is the class of (co)NP problems viz., those that can be solved by non-deterministic algorithms running in polynomial time. The class of (co)NP-complete problems constitutes the (equivalence) class of all the mutually polynomially encodable problems in (co)NP, of which **SAT** is the representative. Hence, **SAT** solvers can be used to solve a wide spectrum of problems.

1. **3-colorability**: We consider the (undirected) graph G_s defined by

   - Vertexes: South American countries.
   - Edges: Any two bordering countries.

   We recall that a graph (V, E) is *k-colorable* if there exists a function $c \colon V \to \{1, \ldots, k\}$ s.t. if $(v, v') \in E$, $c(v) \neq c(v')$.

   The aim of this exercise is to check whether G_s is 3-colorable using **SAT** solving techniques.

2. **(Marked Assignment) Sudoku**: Implement (in the language of your choice, possibly OCaml, a **SAT** solving procedure for Sudoku.